STANDARD MODEL OF THE ELECTROWEAK INTERACTION:

THEORETICAL DEVELOPMENTS

BERND A. KNIEHL[*][†]

Theory Division, National Laboratory for High Energy Physics (KEK)
1-1 Oho, Tsukuba-shi, Ibaraki-ken, 305 Japan

ABSTRACT

We review recent theoretical progress in the computation of radiative corrections beyond one loop within the standard model of electroweak interactions, both in the gauge and Higgs sectors. In the gauge sector, we discuss universal corrections of $\mathcal{O}(G_F^2 M_H^2 M_W^2)$, $\mathcal{O}(G_F^2 m_t^4)$, $\mathcal{O}(\alpha_s G_F M_W^2)$, and those due to virtual $t\bar{t}$ threshold effects, as well as specific corrections to $\Gamma(Z \to b\bar{b})$ of $\mathcal{O}(G_F^2 m_t^4)$, $\mathcal{O}(\alpha_s G_F m_t^2)$, and $\mathcal{O}(\alpha_s^3)$ including finite-m_b effects. We also present an update of the hadronic contributions to $\Delta\alpha$. Theoretical uncertainties, other than those due to the lack of knowledge of M_H and m_t, are estimated. In the Higgs sector, we concentrate on $\Gamma(H \to f\bar{f})$ and consider in $\mathcal{O}(\alpha_s G_F m_t^2)$ the universal corrections and those which are specific for the $b\bar{b}$ mode, as well as $\mathcal{O}(\alpha_s^2)$ corrections in the $q\bar{q}$ channels including the finite-m_q terms.

1. Introduction

As a rule, the size of radiative corrections to a given process is determined by the discrepancy between the various mass and energy scales involved. In Z-boson physics, the dominant effects arise from light charged fermions, which induce large logarithms of the form $\alpha^n \ln^m(M_Z^2/m_f^2)$ ($m \leq n$) in the fine-structure constant (and also in initial-state radiative corrections), and from the top quark, which generates power corrections of the orders $G_F m_t^2$, $G_F^2 m_t^4$, $\alpha_s G_F m_t^2$, etc. On the other hand, the quantum effects due to a heavy Higgs boson are screened, i.e., logarithmic in M_H at one loop and just quadratic at two loops. By contrast, such corrections are proportional to M_H^2 and M_H^4, respectively, in the Higgs sector.

2. Gauge Sector

2.1. Universal Corrections: Electroweak Parameters (Oblique Corrections)

For a wide class of low-energy and Z-boson observables, the dominant effects originate entirely in the gauge-boson propagators (oblique corrections) and may be

parametrized conveniently in terms of four electroweak parameters, $\Delta\alpha$, $\Delta\rho$, Δr, and $\Delta\kappa$, which bear the following physical meanings:[1]

1. $\Delta\alpha$ determines the running fine-structure constant at the Z-boson scale, $\alpha(M_Z)/\alpha = (1 - \Delta\alpha)^{-1}$, where α is the corresponding value at the electron scale;

2. $\Delta\rho$ measures the quantum corrections to the ratio of the neutral- and charged-current amplitudes at low energy,[2] $G_{NC}(0)/G_{CC}(0) = (1 - \Delta\rho)^{-1}$;

3. Δr embodies the non-photonic corrections to the muon lifetime,[3] $G_F = \left(\pi\alpha/\sqrt{2}s_w^2 M_W^2\right)(1 - \Delta r)^{-1}$;

4. $\Delta\kappa$ controls the effective weak mixing angle, $\bar{s}_w^2 = s_w^2(1 + \Delta\kappa)$, that occurs in the ratio of the $f\bar{f}Z$ vector and axial-vector couplings,[4] $v_f/a_f = 1 - 4|Q_f|\bar{s}_w^2$.

Unless stated otherwise, we adopt the on-shell scheme and set[3] $c_w^2 = 1 - s_w^2 = M_W^2/M_Z^2$. The large logarithms are collected by $\Delta\alpha$, and the leading m_t dependence is carried by $\Delta\rho$. Δr and $\Delta\kappa$ may be decomposed as $(1 - \Delta r) = (1 - \Delta\alpha)(1 + c_w^2/s_w^2\Delta\rho) - \Delta r_{rem}$ and $\Delta\kappa = c_w^2/s_w^2\Delta\rho + \Delta\kappa_{rem}$, respectively, where the remainder parts are devoid of m_f logarithms and m_t power terms. The triplet $(\Delta\rho, \Delta r_w, \Delta\kappa)$, where Δr_w is defined by $(1-\Delta r) = (1-\Delta\alpha)(1-\Delta r_w)$, is equivalent to synthetical sets like[5] (S, T, U) and $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$, which have gained vogue recently. We note in passing that the bosonic contributions to these electroweak parameters are, in general, gauge dependent and finite only in a restricted class of gauges if the conventional formulation in terms of vacuum polarizations is employed. This problem may be cured in the framework of the pinch technique.[6]

At two loops, large contributions are expected to arise from the exchange of heavy Higgs bosons, heavy top quarks, and gluons. The hadronic contributions to $\Delta\alpha$ and the $t\bar{t}$ threshold effects on $\Delta\rho$, Δr, and $\Delta\kappa$ cannot be calculated reliably in QCD to finite order. However, they may be related via dispersion relations to data of $e^+e^- \to hadrons$ and theoretical predictions of $e^+e^- \to t\bar{t}$ based on realistic quark potentials, respectively.

2.1.1. Two-Loop $\mathcal{O}(G_F^2 M_H^2 M_Z^2)$ Corrections

Such corrections are generated by two-loop gauge-boson vacuum-polarization diagrams that are constructed from physical and unphysical Higgs bosons. Knowledge[7] of the first two terms of the Taylor expansion around $q^2 = 0$ is sufficient to derive[8,9] the leading contributions to $\Delta\rho$, Δr, and $\Delta\kappa$,

$$\Delta\rho = \frac{G_F^2 M_H^2 M_W^2}{64\pi^4} \frac{s_w^2}{c_w^2} \left(-9\sqrt{3}\,\text{Li}_2\left(\frac{\pi}{3}\right) + \frac{9}{2}\zeta(2) + \frac{9}{4}\pi\sqrt{3} - \frac{21}{8}\right)$$

$$\approx 4.92 \cdot 10^{-5} \left(\frac{M_H}{1\,\text{TeV}}\right)^2, \tag{1}$$

$$\Delta r = \frac{G_F^2 M_H^2 M_W^2}{64\pi^4} \left(9\sqrt{3}\,\text{Li}_2\left(\frac{\pi}{3}\right) - \frac{25}{18}\zeta(2) - \frac{11}{4}\pi\sqrt{3} + \frac{49}{72}\right)$$

$$\approx -1.05 \cdot 10^{-4} \left(\frac{M_H}{1\,\text{TeV}} \right)^2, \tag{2}$$

$$\Delta\kappa = \frac{G_F^2 M_H^2 M_W^2}{64\pi^4} \left(-9\sqrt{3}\,\text{Li}_2\left(\frac{\pi}{3} \right) + \frac{53}{18}\zeta(2) + \frac{5}{2}\pi\sqrt{3} - \frac{119}{72} \right)$$

$$\approx 1.37 \cdot 10^{-4} \left(\frac{M_H}{1\,\text{TeV}} \right)^2, \tag{3}$$

Due to the smallness of the prefactors, these contributions are insignificant for $M_H \lesssim 1$ TeV.

2.1.2. Two-Loop $\mathcal{O}(G_F^2 m_t^4)$ Corrections for $M_H \neq 0$

Also at two loops, $\Delta\rho$ picks up the leading large-m_t term, and Δr and $\Delta\kappa$ depend on m_t chiefly via $\Delta\rho$. Neglecting m_b and defining $x_t = \left(G_F m_t^2 / 8\pi^2 \sqrt{2} \right)$, one has

$$\Delta\rho = 3x_t \left[1 + x_t \rho^{(2)} \left(\frac{M_H}{m_t} \right) - \frac{2}{3}(2\zeta(2) + 1)\frac{\alpha_s(m_t)}{\pi} \right], \tag{4}$$

where, for completeness, also the well-known $\mathcal{O}(\alpha_s G_F m_t^2)$ term[10,11] is included. Very recently, also the $\mathcal{O}(\alpha_s^2 G_F m_t^2)$ term has been computed,[12] the result being $(-21.27063 + 1.78621\,N_F)(\alpha_s/\pi)^2$, where N_F is the number of active quark flavours; the details are reported elsewhere.[13] The coefficient $\rho^{(2)}(r)$ is negative for all plausible values of r, bounded from below by $\rho^{(2)}(5.72) = -11.77$, and exhibits the following asymptotic behaviour:[14,15]

$$\rho^{(2)}(r) = \begin{cases} -12\zeta(2) + 19 - 4\pi r + \mathcal{O}(r^2 \ln r), & \text{if } r \ll 1; \\ 6\ln^2 r - 27\ln r + 6\zeta(2) + \frac{49}{4} + O\left(\frac{\ln^2 r}{r^2} \right), & \text{if } r \gg 1. \end{cases} \tag{5}$$

The value at[16] $r = 0$ greatly underestimates the effect. Both $\mathcal{O}(G_F^2 m_t^4)$ and $\mathcal{O}(\alpha_s G_F m_t^2)$ corrections screen the one-loop result and thus increase the value of m_t predicted indirectly from global analyses of low-energy, M_W, LEP/SLC, and other high-precision data. Recently, a first attempt was made to control subleading corrections to $\Delta\rho$, of $\mathcal{O}\left(G_F^2 m_t^2 M_Z^2 \ln(M_Z^2/m_t^2) \right)$, in an SU(2) model of weak interactions, and significant effects were found.[15]

2.1.3. Two-Loop $\mathcal{O}(\alpha_s G_F M_W^2)$ Corrections

For $m_t \gg M_W$, the bulk of the QCD corrections is concentrated in $\Delta\rho$; see Eq. (4). However, for realistic values of m_t, the subleading terms, of $\mathcal{O}(\alpha_s G_F M_W^2)$, are significant numerically, e.g., they amount to 20% of the full two-loop QCD correction to Δr at $m_t = 150$ GeV. Specifically, one has[8,11]

$$\Delta r_{rem} = \frac{G_F M_W^2}{\pi^3 \sqrt{2}} \left\{ -\alpha_s(M_Z)\left(\frac{c_w^2}{s_w^2} - 1 \right) \ln c_w^2 \right.$$

$$\left. + \alpha_s(m_t)\left[\left(\frac{1}{3} - \frac{1}{4s_w^2} \right) \ln \frac{m_t^2}{M_Z^2} + A + \frac{B}{s_w^2} \right] \right\}, \tag{6}$$

$$\Delta\kappa_{rem} = \frac{G_F M_W^2}{\pi^3\sqrt{2}}\left\{\alpha_s(M_Z)\frac{c_w^2}{s_w^2}\ln c_w^2 - \alpha_s(m_t)\left[\left(\frac{1}{6}-\frac{1}{4s_w^2}\right)\ln\frac{m_t^2}{M_Z^2}+\frac{A}{2}+\frac{B}{s_w^2}\right]\right\}, \quad (7)$$

where terms of $\mathcal{O}(M_Z^2/m_t^2)$ are omitted within the square brackets and

$$A = \frac{1}{3}\left(-4\zeta(3)+\frac{4}{3}\zeta(2)+\frac{5}{2}\right) \approx -0.03833, \quad (8)$$

$$B = \zeta(3) - \frac{2}{9}\zeta(2) - \frac{1}{4} \approx 0.58652. \quad (9)$$

For contributions due to the tb doublet, $\mu = m_t$ is the natural scale for $\alpha_s(\mu)$.

2.1.4. Hadronic Contributions to $\Delta\alpha$

Jegerlehner has updated his 1990 analysis[17] of the hadronic contributions to $\Delta\alpha$ by taking into account the hadronic resonance parameters specified in the 1992 report[18] by the Particle Data Group and recently published low-energy e^+e^- data taken at Novosibirsk. The (preliminary) result at $\sqrt{s} = 91.175$ GeV reads[19]

$$\Delta\alpha_{hadrons} = 0.0283 \pm 0.0007, \quad (10)$$

i.e., the central value has increased by $1\cdot 10^{-4}$, while the error has decreased by $\pm 2\cdot 10^{-4}$. The latter is particularly important, since this error has long constituted the dominant uncertainty for theoretical predictions of electroweak parameters. For comparison, we list the leptonic contribution up to two loops in QED,[11]

$$\Delta\alpha_{leptons} = \frac{\alpha}{3\pi}\sum_\ell\left[\ln\frac{M_Z^2}{m_\ell^2}-\frac{5}{3}+\frac{\alpha}{\pi}\left(\frac{3}{4}\ln\frac{M_Z^2}{m_\ell^2}+3\zeta(2)-\frac{5}{8}\right)+O\left(\frac{m_\ell^2}{M_Z^2}\right)\right]$$
$$= 0.031\,496\,6 \pm 0.000\,000\,4, \quad (11)$$

where the error stems from the current m_τ world average,[20] $m_\tau = (1777.0\pm0.4)$ MeV.

2.1.5. $t\bar{t}$ Threshold Effects

Although loop amplitudes involving the top quark are mathematically well behaved, it is evident that interesting and possibly significant features connected with the $t\bar{t}$ threshold cannot be accommodated when the perturbation series is truncated at finite order. In fact, perturbation theory up to $\mathcal{O}(\alpha\alpha_s)$ predicts a discontinuous steplike threshold behaviour for $\sigma(e^+e^-\to t\bar{t})$. A more realistic description includes the formation of toponium resonances by multi-gluon exchange. For $m_t \gtrsim 130$ GeV, the revolution period of a $t\bar{t}$ bound state exceeds its lifetime, and the individual resonances are smeared out to a coherent structure. By Cutkosky's rule, $\sigma(e^+e^-\to t\bar{t})$ corresponds to the absorptive parts of the photon and Z-boson vacuum polarizations, and its enhancement at threshold induces additional contributions in the corresponding real parts, which can be computed via dispersive techniques. Decomposing the vacuum-polarization tensor generated by the insertion of a top-quark loop into a gauge-boson line as

$$\Pi_{\mu\nu}^{V,A}(q) = \Pi^{V,A}(q^2)g_{\mu\nu} + \lambda^{V,A}(q^2)q_\mu q_\nu, \quad (12)$$

where V and A label the vector and axial-vector components and q is the external four-momentum, and imposing Ward identities, one derives the following set of dispersion relations:[21]

$$\Pi^V(q^2) = \frac{q^2}{\pi} \int \frac{ds}{s} \frac{\operatorname{Im}\Pi^V(s)}{q^2 - s - i\epsilon}, \tag{13}$$

$$\Pi^A(q^2) = \frac{1}{\pi} \int ds \left(\frac{\operatorname{Im}\Pi^A(s)}{q^2 - s - i\epsilon} + \operatorname{Im}\lambda^A(s) \right). \tag{14}$$

The alternative set of dispersion relations proposed in Ref. 22 does not, in general, yield correct results, as has been demonstrated[23] by establishing a perturbative counterexample, namely the $\mathcal{O}(\alpha_s G_F m_t^2)$ corrections to $\Gamma(H \to \ell^+\ell^-)$ (see Sect. 3.1.). It has been suggested that this argument may be extended to all orders in α_s by means of the operator product expansion.[24] In the threshold region, only $\operatorname{Im}\Pi^V(q^2)$ and $\operatorname{Im}\lambda^A(q^2)$ receive significant contributions and are related by $\operatorname{Im}\lambda^A(q^2) \approx -\operatorname{Im}\Pi^V(q^2)/q^2$, while $\operatorname{Im}\Pi^A(q^2)$ is strongly suppressed due to centrifugal barrier effects.[21] Of course,[21] $\lambda^V(q^2) = -\Pi^V(q^2)/q^2$. These contributions in turn lead to shifts in $\Delta\rho$, Δr, and $\Delta\kappa$. A crude estimation may be obtained by setting $\operatorname{Im}\Pi^V(q^2) = \operatorname{Im}\Pi^V(4m_t^2) = \alpha_s m_t^2$ in the interval $(2m_t - \Delta)^2 \leq q^2 \leq 4m_t^2$, where Δ may be regarded as the binding energy of the 1S state. This yields

$$\Delta\rho = -\frac{G_F}{2\sqrt{2}} \frac{\alpha_s}{\pi} m_t \Delta, \tag{15}$$

$$\Delta r = -\frac{c_w^2}{s_w^2} \Delta\rho \left[1 - \left(1 - \frac{8}{3}s_w^2\right)^2 \frac{M_Z^2}{4m_t^2 - M_Z^2} + \frac{16}{9} s_w^4 \frac{M_Z^2}{m_t^2} \right], \tag{16}$$

$$\Delta\kappa = \frac{c_w^2}{s_w^2} \Delta\rho \left[1 - \left(1 - \frac{8}{3}s_w^2\right) \frac{M_Z^2}{4m_t^2 - M_Z^2} \right]. \tag{17}$$

Obviously, the threshold effects have the same sign as the $\mathcal{O}(\alpha_s G_F m_t^2)$ corrections. For realistic quark potentials, one has approximately $\Delta \propto m_t$, so that the threshold contributions scale like m_t^2. Again, $\Delta\rho$ is most strongly affected, while the corrections to Δr_{rem} and $\Delta\kappa_{rem}$ are suppressed by M_Z^2/m_t^2. A comprehensive numerical analysis may be found in Refs. 21,25,26. For 150 GeV $\leq m_t \leq$ 200 GeV, the threshold effects enhance the QCD corrections by roughly 30%.

We emphasize that the above QCD corrections come with both experimental and theoretical errors. The experimental errors are governed by the α_s measurement,[27] $\alpha_s(M_Z) = 0.118 \pm 0.006$. Assuming $m_t = 174$ GeV, this amounts to errors of $\pm 5\%$ and $\pm 18\%$ on the continuum and threshold contributions to $\Delta\rho$, respectively. This reflects the fact the α_s dependence is linear in the continuum, while that of 1S peak height is approximately cubic. Theoretical errors are due to unknown higher-order corrections. In the continuum, they are usually estimated by varying the renormalization scale, μ, of $\alpha_s(\mu)$ in the range $m_t/2 \leq \mu \leq 2m_t$, which amounts to $\pm 11\%$. The theoretical error on the threshold contribution is mainly due to model dependence and is estimated to be $\pm 20\%$ by comparing conventional quark

potentials. A conservative analysis of the combined error on the absolute value of $\Delta\rho$ at $m_t = 174$ GeV yields $\pm 1.5 \cdot 10^{-4}$. Due to the magnification factor c_w^2/s_w^2, the corresponding error on Δr and $\Delta\kappa$ is $\pm 5.0 \cdot 10^{-4}$. We stress that, in the case of Δr and thus the M_W prediction from the muon lifetime, this error is almost as large as the one from hadronic sources introduced via $\Delta\alpha$; see Eq. (10). For higher m_t values, it may even be larger.

In Eq. (4), we have evaluated the $\mathcal{O}(\alpha_s G_F m_t^2)$ correction at $\mu = m_t$, since this is the only scale available. However, this is a leading-order QCD prediction, which suffers from the usual scale ambiguity. We may choose $\mu = \xi m_t$ in such a way that the $\mathcal{O}\!\left(\alpha_s(\mu)G_F m_t^2\right)$ calculation agrees with the $\mathcal{O}\!\left(\alpha_s(m_t)G_F m_t^2\right)$ one plus the $t\bar{t}$ threshold effects. In the case of $\Delta\rho$, this leads to $\xi = 0.190^{+0.097}_{-0.057}$, where we have included the $\pm 30\%$ error on the $t\bar{t}$ threshold contribution. Alternative, conceptually very different approaches of scale setting[28,29,30] yield results in the same ball park. In Ref. 28, it is suggested that long-distance effects lower the renormalization point for $\alpha_s(\mu)$ in Eq. (4) through the contributions of the near-mass-shell region to the evolution of the quark mass from the mass shell to distances of order $1/m_t$. To estimate these effects, the authors of Ref. 28 apply the Brodsky-Lepage-Mackenzie (BLM) criterion[31] to Eq. (4) and find $\xi = 0.154$. The author of Ref. 29 expresses first the fermionic contribution to $\Delta\rho$ in terms of $\overline{m}_t(m_t)$, where $\overline{m}_t(\mu)$ is the top-quark $\overline{\text{MS}}$ mass at renormalization scale μ, and then relates $\overline{m}_t(m_t)$ to m_t by optimizing the expansion of $m_t/\overline{m}_t(m_t)$, which is known through $\mathcal{O}(\alpha_s^2)$,[32] according to the BLM criterion.[31] In Ref. 30, he refines this argument by using the new results of Ref. 12 and an expansion of $\mu_t/\overline{m}_t(m_t)$, where $\mu_t = \overline{m}_t(\mu_t)$, and obtains $\xi = 0.323$. Finally, we observe that the $\mathcal{O}(\alpha_s^2 G_F m_t^2)$ term indeed has the very sign predicted by the study[21,25,26] of the $t\bar{t}$ threshold effects and accounts also for the bulk of their size. In fact, this term may be absorbed into the $\mathcal{O}(\alpha_s G_F m_t^2)$ term by choosing[12] $\xi = 0.348$ for $N_F = 6$. Arguing that $N_F = 5$ is more appropriate for $\mu < m_t$, this value comes down to[12] $\xi = 0.324$, which is not far outside the range $0.133 \leq \xi \leq 0.287$ predicted from the $t\bar{t}$ threshold analysis. The residual difference may be understood by observing that the ladder diagrams of $\mathcal{O}(\alpha_s^n G_F m_t^2)$, with $n \geq 3$, are not included in the fixed-order calculation of Ref. 12.

The claim[33] that the $t\bar{t}$ threshold effects are greatly overestimated in Refs. 21,25 is based on a simplified analysis, which demonstrably[26] suffers from a number of severe analytical and numerical errors. Speculations[34] that the dispersive computation of $t\bar{t}$ threshold effects is unstable are quite obviously unfounded, since they arise from uncorrelated and unjustifiably extreme variations of the continuum and threshold contributions. In particular, the authors of Ref. 34 ascribe the unavoidable scale dependence of the $\mathcal{O}(\alpha_s G_F m_t^2)$ continuum result to the uncertainty in the much smaller threshold contribution, which artificially amplifies this uncertainty. In fact, the sum of both contributions, which is the physically relevant quantity, is considerably less μ dependent than the continuum contribution alone.[26]

2.2. Specific Corrections: $\Gamma\left(Z \to b\bar{b}\right)$ and $\Gamma(Z \to \text{hadrons})$

The observable $\Gamma\left(Z \to b\bar{b}\right)$ deserves special attention, since it receives specific m_t power corrections. These may be accommodated in the improved Born approximation[4,8] by replacing the parameters $\rho = (1 - \Delta\rho)^{-1}$ and $\kappa = 1 + \Delta\kappa$ by $\rho_b = \rho(1+\tau)^2$ and $\kappa_b = \kappa(1+\tau)^{-1}$, respectively, where τ is an additional electroweak parameter. Similarly to $\Delta\rho$, τ receives contributions in the orders $G_F m_t^2$, $G_F^2 m_t^4$, $\alpha_s G_F m_t^2$, etc.

2.2.1. Two-Loop $\mathcal{O}(G_F^2 m_t^4)$ Corrections for $M_H \neq 0$

In the oblique corrections considered so far, the m_t dependence might be masked by all kinds of physics beyond the standard model. Contrariwise, in the case of $Z \to b\bar{b}$, the virtual top quark is tagged directly by the external bottom flavour. At one loop, there is a strong cancellation between the flavour-independent oblique corrections, $\Delta\rho$ and $\Delta\kappa$, and the specific $Z \to b\bar{b}$ vertex correction,[35] τ.

The leading two-loop corrections to τ, of[14] $\mathcal{O}(G_F^2 m_t^4)$ and[36] $\mathcal{O}(\alpha_s G_F m_t^2)$, have recently become available. The master formula reads

$$\tau = -2x_t \left(1 + x_t \tau^{(2)}\left(\frac{M_H}{m_t}\right) - 2\zeta(2)\frac{\alpha_s(m_t)}{\pi}\right),\tag{18}$$

where x_t is defined above Eq. (4). $\tau^{(2)}(r)$ rapidly varies with r, $\tau^{(2)}(r) \geq \tau^{(2)}(1.55) = 1.23$, and its asymptotic behaviour is given by[14]

$$\tau^{(2)}(r) = \begin{cases} -2\zeta(2) + 9 - 4\pi r + \mathcal{O}(r^2 \ln r), & \text{if } r \ll 1; \\ \dfrac{5}{2}\ln^2 r - \dfrac{47}{12}\ln r + \zeta(2) + \dfrac{311}{144} + O\left(\dfrac{\ln^2 r}{r^2}\right), & \text{if } r \gg 1. \end{cases}\tag{19}$$

The value at $r = 0$ has been confirmed by a third group.[37]

2.2.2. Two-Loop $\mathcal{O}(\alpha_s G_F m_t^2)$ Corrections

In Eq. (18), we have also included the $\mathcal{O}(\alpha_s G_F m_t^2)$ term,[36] assuming that the formula for $\Gamma\left(Z \to b\bar{b}\right)$ is, at the same time, multiplied by the overall factor $(1+\alpha_s/\pi)$, which is the common beginning of the QCD perturbation series of the quark vector and axial-vector current correlators, R^V and R^A. We observe that the $\mathcal{O}(G_F^2 m_t^4)$ and $\mathcal{O}(\alpha_s G_F m_t^2)$ terms of Eq. (18) cancel partially.

2.2.3. Three-Loop $\mathcal{O}(\alpha_s^3)$ Corrections

Most of the results discussed in this section are valid also for the $Z \to q\bar{q}$ decays with $q \neq b$. Here, we put $m_q = 0$, except for $q = t$. Finite-m_q effects will be considered in the next section. By the optical theorem, the QCD corrections to $\Gamma(Z \to q\bar{q})$ may be viewed as the imaginary parts of the Z-boson self-energy diagrams that contain a q-quark loop decorated with virtual gluons and possibly other quark loops. Diagrams where the two Z-boson lines are linked to the same quark loop are usually called non-singlet, while the residual diagrams are called singlet, which includes the so-called double-triangle diagrams. By γ_5 reflection, the

non-singlet contribution, R_{NS}, to R^A coincides with the one to R^V. Up to $\mathcal{O}(\alpha_s^3)$ in the $\overline{\text{MS}}$ scheme with $N_F = 5$, one has[38]

$$R_{NS} = 1 + \frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2 \left(1.40923 + F\left(\frac{M_Z}{4m_t^2}\right)\right) - 12.76706 \left(\frac{\alpha_s}{\pi}\right)^3. \tag{20}$$

F collects the decoupling-top-quark effects in $\mathcal{O}(\alpha_s^2)$ and has the expansion[39]

$$F(r) = r\left[-\frac{8}{135}\ln(4r) + \frac{176}{675}\right] + \mathcal{O}(r^2). \tag{21}$$

F has also been obtained in numerical form recently.[40] We note that an analytic expression for F had been known previously from the study of the two-loop QED vertex correction due to virtual heavy fermions.[41] Recently, the $\mathcal{O}(\alpha_s^4)$ term of Eq. (20) has been estimated using the principle of minimal sensitivity and the effective-charges approach.[42] The $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha\alpha_s)$ corrections to $\Gamma\left(Z \to b\bar{b}\right)$ from photonic source are well under control.[43]

Due to Furry's theorem, singlet diagrams with $q\bar{q}Z$ vector couplings occur just in $\mathcal{O}(\alpha_s^3)$. They contain two quark loops at the same level of hierarchy, which, in general, involve different flavours. Thus, they cannot be assigned unambiguously to a specific $q\bar{q}$ channel. In practice, this does not create a problem, since their combined contribution to $\Gamma(Z \to \text{hadrons})$ is very small anyway,[38]

$$\delta\Gamma_Z = \frac{G_F M_Z^3}{8\pi\sqrt{2}} \left(\sum_{q=u,d,s,c,b} v_q\right)^2 (-0.41318) \left(\frac{\alpha_s}{\pi}\right)^3, \tag{22}$$

where $v_q = 2I_q - 4Q_q s_w^2$.

Axial-type singlet diagrams contribute already in $\mathcal{O}(\alpha_s^2)$. The sum over triangle subgraphs involving mass-degenerate (e.g., massless) up- and down-type quarks vanishes. Thus, after summation, only the double-triangle diagrams involving t and b quarks contribute to $\Gamma\left(Z \to b\bar{b}\right)$ and $\Gamma(Z \to \text{hadrons})$. The present knowledge of the singlet part, R_S^A, of R^A is summarized by (m_t is the top-quark pole mass)

$$R_S^A = \left(\frac{\alpha_s}{\pi}\right)^2 \frac{1}{3} I\left(\frac{M_Z^2}{4m_t^2}\right) + \left(\frac{\alpha_s}{\pi}\right)^3 \left(\frac{23}{12}\ln^2\frac{m_t^2}{M_Z^2} - \frac{67}{18}\ln\frac{m_t^2}{M_Z^2} - 15.98773\right). \tag{23}$$

An analytic expression for the I function may be found in Ref. 44; its high-m_t expansion reads[44]

$$I(r) = 3\ln(4r) - \frac{37}{4} + \frac{28}{27}r + \mathcal{O}(r^2). \tag{24}$$

The second term on the right-hand side of Eq. (24) has been confirmed recently.[45] The $\mathcal{O}(\alpha_s^3)$ logarithmic terms of Eq. (23) follow from Eq. (24) by means of renormali-

zation-group techniques,[45] while the constant term requires a separate computation.[46]

2.2.4. Finite-m_b Effects

In $\mathcal{O}(\alpha_s)$, the full m_b dependence of R^V and R^A is known,[22,47] while, in higher orders, only the first terms of their m_b^2/M_Z^2 expansions have been calculated.[48,49,50] In the $\overline{\text{MS}}$ scheme, one has

$$\delta R^V = \frac{12\overline{m}_b^2}{M_Z^2}\frac{\alpha_s}{\pi}\left[1 + \frac{629}{72}\frac{\alpha_s}{\pi} + 45.14610\left(\frac{\alpha_s}{\pi}\right)^2\right], \tag{25}$$

$$\delta R^A = -\frac{6\overline{m}_b^2}{M_Z^2}\left[1 + \frac{11}{3}\frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2\left(11.28560 - \ln\frac{m_t^2}{M_Z^2}\right)\right], \tag{26}$$

where α_s and the b-quark $\overline{\text{MS}}$ mass, $\overline{m}_b$, are to be evaluated at $\mu = M_Z$. The second and third terms of Eq. (25) come from Refs. 48,49, respectively, and the third term of Eq. (26) is from Ref. 50. Due to the use of $\overline{m}_b(M_Z)$, Eqs. (25,26) are devoid of terms involving $\ln(M_Z^2/m_b^2)$. The $\mathcal{O}(\alpha_s m_b^2/M_Z^2)$ corrections should be detectable. The finite-m_b terms beyond $\mathcal{O}(\alpha_s)$ in Eqs. (25,26) each amount to approximately $5 \cdot 10^{-3}\%$ of $\Gamma\left(Z \to b\bar{b}\right)$ but have opposite signs.

3. Higgs Sector: Corrections to $\Gamma\left(H \to f\bar{f}\right)$

Quantum corrections to Higgs-boson phenomenology have received much attention in the literature; for a review, see Ref. 51. The experimental relevance of radiative corrections to the $f\bar{f}$ branching fractions of the Higgs boson has been emphasized recently in the context of a study[52] dedicated to LEP 2. Techniques for the measurement of these branching fractions at a $\sqrt{s} = 500$ GeV e^+e^- linear collider have been elaborated in Ref. 53.

In the Born approximation, the $f\bar{f}$ partial widths of the Higgs boson are given by

$$\Gamma_0\left(H \to f\bar{f}\right) = \frac{N_f G_F M_H m_f^2}{4\pi\sqrt{2}}\left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2}, \tag{27}$$

where $N_f = 1$ (3) for lepton (quark) flavours.

The full one-loop electroweak corrections to Eq. (27) are now well established.[54,55] They consist of an electromagnetic and a weak part, which are separately finite and gauge independent. They may be included in Eq. (27) as an overall factor, $\left[1 + (\alpha/\pi)Q_f^2\Delta_{em}\right](1 + \Delta_{weak})$. For $M_H \gg 2m_f$, Δ_{em} develops a large logarithm,

$$\Delta_{em} = -\frac{3}{2}\ln\frac{M_H^2}{m_f^2} + \frac{9}{4} + \mathcal{O}\left(\frac{m_f^2}{M_H^2}\ln\frac{M_H^2}{m_f^2}\right). \tag{28}$$

For $M_H \ll 2M_W$, the weak part is well approximated by[55]

$$\Delta_{weak} = \frac{G_F}{8\pi^2\sqrt{2}} \left\{ C_f m_t^2 + M_W^2 \left(\frac{3}{s_w^2} \ln c_w^2 - 5 \right) + M_Z^2 \left[\frac{1}{2} - 3 \left(1 - 4s_w^2 |Q_f| \right)^2 \right] \right\},$$
(29)

where $C_b = 1$ and $C_f = 7$ for all other flavours, except for top. The $t\bar{t}$ mode will not be probed experimentally anytime soon and we shall not be concerned with it in the remainder of this presentation. From Eq. (29) it is evident that the dominant effect is due to virtual top quarks. In the case $f \neq b$, the m_t dependence is carried solely by the renormalizations of the wave function and the vacuum expectation value of the Higgs field and is thus flavour independent. These corrections are of the same nature as those considered in Ref. 56. For $f = b$, there are additional m_t-dependent contributions from the $b\bar{b}H$ vertex correction and the b-quark wave-function renormalization. Incidentally, they cancel almost completely the universal m_t dependence. It is amusing to observe that a similar situation has been encountered in the context of the $Z \to f\bar{f}$ decays.[35] The QCD corrections to the universal and non-universal $\mathcal{O}(G_F m_t^2)$ terms will be presented in the next two sections.

3.1. Two-Loop $\mathcal{O}(\alpha_s G_F m_t^2)$ Universal Corrections

The universal $\mathcal{O}(G_F m_t^2)$ term of Δ_{weak} resides inside the combination

$$\Delta_u = -\frac{\Pi_{WW}(0)}{M_W^2} - \text{Re}\,\Pi'_{HH}\left(M_H^2\right),$$
(30)

where Π_{WW} and Π_{HH} are the unrenormalized self-energies of the W and Higgs bosons, respectively.[55] The same is true of its QCD correction.

For $M_H < 2m_t$ and $m_b = 0$, the one-loop term reads[55]

$$\Delta_u^0 = 4N_c x_t \left[\left(1 + \frac{1}{2r} \right) \sqrt{\frac{1}{r} - 1} \arcsin \sqrt{r} - \frac{1}{4} - \frac{1}{2r} \right],$$
(31)

where $r = (M_H^2/4m_t^2)$ and x_t is defined above Eq. (4). In the same approximation, the two-loop term may be written as[23,57]

$$\Delta_u^1 = N_c C_F x_t \frac{\alpha_s}{\pi} \left(6\zeta(3) + 2\zeta(2) - \frac{19}{4} - \text{Re}\,H'_1(r) \right),$$
(32)

where $C_F = (N_c^2 - 1)/(2N_c) = 4/3$ and H_1 has an expression in terms of dilogarithms and trilogarithms.[57] Equation (32) has been confirmed recently.[58] In the heavy-quark limit ($r \ll 1$), one has[57]

$$H'_1(r) = 6\zeta(3) + 3\zeta(2) - \frac{13}{4} + \frac{122}{135}r + \mathcal{O}(r^2).$$
(33)

Combining Eqs. (31,32) and retaining only the leading high-m_t terms, one finds the QCD-corrected coefficients C_f for $f \neq b$,

$$C_f = 7 - 2\left(\frac{\pi}{3} + \frac{3}{\pi}\right)\alpha_s \approx 7 - 4.00425\,\alpha_s. \tag{34}$$

This result has been reproduced recently.[59] We recover the notion that, in electroweak physics, the one-loop $\mathcal{O}\left(G_F m_t^2\right)$ terms get screened by their QCD corrections. The QCD correction to the shift in $\Gamma\left(H \to f\bar{f}\right)$ induced by a pair of novel quarks with arbitrary masses may be found in Ref. 57.

3.2. Two-Loop $\mathcal{O}(\alpha_s G_F m_t^2)$ Non-universal Corrections

The QCD correction to the non-universal one-loop contribution to $\Gamma\left(H \to b\bar{b}\right)$ arises in part from genuine two-loop three-point diagrams, which are more involved technically. However, the leading high-m_t term may be extracted[60] by means of a low-energy theorem,[61] which relates the amplitudes of two processes that differ by the insertion of an external Higgs-boson line carrying zero momentum. In this way, one only needs to compute the irreducible two-loop b-quark self-energy diagrams with one gluon and one longitudinal W boson, which may be taken massless. After using the Dirac equation and factoring out one power of m_b, one may put $m_b = 0$ in the two-loop integrals, which may then be solved analytically. Applying the low-energy theorem and performing on-shell renormalization, one eventually finds the non-universal leading high-m_t term along with its QCD correction,[60]

$$\Delta_{nu} = x_t \left(-6 + \frac{3}{2} C_F \frac{\alpha_s}{\pi}\right). \tag{35}$$

Combining the term contained within the parentheses with Eq. (34), one obtains the QCD-corrected coefficient C_b,

$$C_b = 1 - 2\left(\frac{\pi}{3} + \frac{2}{\pi}\right)\alpha_s \approx 1 - 3.36763\,\alpha_s. \tag{36}$$

Again, the $\mathcal{O}(G_F m_t^2)$ term is screened by its QCD correction.

3.3. Two-Loop $\mathcal{O}(\alpha_s^2)$ Corrections Including Finite-m_q Effects

In the on-shell scheme, the one-loop QCD correction[62] to $\Gamma(H \to q\bar{q})$ emerges from one-loop QED correction by substituting $\alpha_s C_F$ for αQ_f^2. From Eq. (28) it is apparent that, for $m_q \ll M_H/2$, large logarithmic corrections occur. In general, they are of the form $(\alpha_s/\pi)^n \ln^m(M_H^2/m_q^2)$, with $n \geq m$. Owing to the renormalization-group equation, these logarithms may be absorbed completely into the running $\overline{\text{MS}}$ quark mass, $\overline{m}_q(\mu)$, evaluated at $\mu = M_H$. A similar mechanism has been exploited also in Eqs. (25,26). In this way, these logarithms are resummed to all orders and

the perturbation expansion converges more rapidly. This observation gives support to the notion that the $q\bar{q}H$ Yukawa couplings are controlled by the running quark masses.

For $q \neq t$, the QCD corrections to $\Gamma\left(H \to q\bar{q}\right)$ are known up to $\mathcal{O}(\alpha_s^2)$. In the $\overline{\text{MS}}$ scheme, the result is[51,63]

$$
\Gamma\left(H \to q\bar{q}\right) = \frac{3 G_F M_H \overline{m}_q^2}{4\pi\sqrt{2}} \left[\left(1 - 4\frac{\overline{m}_q^2}{M_H^2}\right)^{3/2} + C_F \frac{\alpha_s}{\pi} \left(\frac{17}{4} - 30\frac{\overline{m}_q^2}{M_H^2}\right) \right.
$$
$$
\left. + \left(\frac{\alpha_s}{\pi}\right)^2 \left(K_1 + K_2 \frac{\overline{m}_q^2}{M_H^2} + 12 \sum_{i=u,d,s,c,b} \frac{\overline{m}_i^2}{M_H^2}\right) \right], \qquad (37)
$$

where $K_1 = 35.93996 - 1.35865\,N_F$,[64] $K_2 = -129.72924 + 6.00093\,N_F$,[63] with N_F being the number of quark flavours active at $\mu = M_H$, and it is understood that α_s, $\overline{m}_q$, and $\overline{m}_i$ are to be evaluated at this scale.

The electroweak corrections may be implemented in Eq. (37) by multiplication with $\left[1 + (\alpha/\pi)Q_f^2\Delta_{em}\right](1 + \Delta_{weak})$, where Δ_{em} and Δ_{weak} are given in Eqs. (28,29), respectively. To include also the $\mathcal{O}(\alpha_s G_F m_t^2)$ corrections, one substitutes in Eq. (29) the QCD-corrected C_f terms specified in Eqs. (34,36). We note in passing that our result[60] disagrees with a recent calculation[59] of the $\mathcal{O}(\alpha_s G_F m_t^2)$ correction to $\Gamma\left(H \to b\bar{b}\right)$ in the on-shell scheme.

4. Conclusions

In conclusion, all dominant two-loop and even certain three-loop radiative corrections to Z-boson physics are now available. However, one has to bear in mind that, apart from the lack of knowledge of the accurate values of M_H and m_t, the reliability of the theoretical predictions is limited by a number of error sources. The inherent QCD errors on the hadronic contribution to $\Delta\alpha$ and the tb contribution to $\Delta\rho$ are $\delta\Delta\alpha = \pm 7 \cdot 10^{-4}$ and $\delta\Delta\rho = \pm 1.5 \cdot 10^{-4}$, respectively, which amounts to $\delta\Delta r = \pm 8.6 \cdot 10^{-4}$. The unknown electroweak corrections are of the order $(\alpha/\pi s_w^2)^2(m_t^2/M_Z^2)\ln(m_t^2/M_Z^2) \approx 6 \cdot 10^{-4}$, possibly multiplied by a large prefactor.[15] The scheme dependence of the key electroweak parameters has been estimated in Refs. 25,65,66 by comparing the evaluations in the on-shell scheme and certain variants of the $\overline{\text{MS}}$ scheme; the maximum variation of Δr in the ranges $60\ \text{GeV} < M_H < 1\ \text{TeV}$ and $150\ \text{GeV} < m_t < 200\ \text{GeV}$ is $8 \cdot 10^{-5}$ when the coupling-constant renormalization is converted[25] and $4 \cdot 10^{-4}$ when the top-quark mass is redefined taking into account just the QCD corrections.[65] The effect on $\Delta\rho$ of including also the leading electroweak corrections in the redefinition of the top-quark mass has been investigated[66] recently in the approximation $M_H, m_t \gg M_Z$. The theoretical predictions for Higgs-boson physics at present and near-future colliding-beam experiments are probably far more precise than the expected theoretical errors.

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6. References

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